

S.-T. Yau College Student Mathematics Contest, 2018
Applied Mathematics, team

1. Given a set of column vectors $y_1, \dots, y_n \in \mathcal{R}^m$, we set $\mathcal{V} = \text{span}\{y_1, \dots, y_n\} \subset \mathcal{R}^m$. How to find $\ell \leq \dim \mathcal{V}$ orthonormal vectors $\{\psi_i\}_{i=1}^\ell$ in \mathcal{R}^m that minimize

$$J(\psi_1, \dots, \psi_\ell) = \sum_{j=1}^n \left\| y_j - \sum_{i=1}^{\ell} (y_j^T \psi_i) \psi_i \right\|^2$$

with the Euclidean norm $\|y\| = \sqrt{y^T y}$.

2. Suppose there are n hyper-planes in the d dimensional Euclidean space \mathbb{R}^d , the planes partition the space into convex cells, the maximal number of cells is denoted as $f_d(n)$,
1. Find the formula for $d = 2$ case, namely, the plane is partitioned by n lines, prove it.
 2. Find the formula for general d and n and prove.

